Novel Distance and Similarity Measure on Pythagorean Fuzzy Sets and its Application to Multicriteria Decision Making with ELECTRE Method for Selection of Best Mobile Phones

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Abstract:

The idea of Pythagorean fuzzy sets (PFSs) was originally given by Yager in 2013, to handle incomplete and indistinct information in everyday life more effectively with reasonable accuracy than Fuzzy sets and Intuitionistic fuzzy sets, respectively. Distance and similarity measures are dual concepts which are used to check the difference and resemblance respectively between PFSs. As far as our knowledge is concerned, no one has transformed PFSs to intervalued Pythagorean fuzzy sets (IVPFSs) so far. So, in this article, we suggest a novel distance and similarity measure between Pythagorean fuzzy sets based on the transformation from PFSs to IVPFSs. We construct axiomatic definition based on proposed distance and similarity measure between IVPFSs. Furthermore, we present some examples interconnected to pattern recognition and multicriteria decision making (MCDM) to exhibit the accuracy and applicability of our proposed distance and similarity measure. We construct an algorithm for ELECTRE (Elimination and Choice Expressing Reality) method using our proposed method to handle complex multicriteria decision-making problems related to daily life. Our final results exhibit that the suggested method is rational and authentic to treat different complex problems linked to our everyday life situation especially in MCDM situations.

Keywords: Pythagorean fuzzy sets, Inter valued Pythagorean fuzzy sets, Distance and Similarity measures, Pattern recognition, ELECTRE, Multi-criteria decision making.

1. Introduction

The general idea of fuzzy sets is one of the major tools in development of thinking computer systems. Fuzzy sets can determine a wide series of difficulty of control, planning, reasoning, computer vision and pattern classification. Meanwhile crisp set utilizes characteristic function that allocates a value of either 0 or 1 to every component of the universe. In the theory of fuzzy sets, we frequently mention crisp sets as "ordinary set". The concept of fuzzy sets (FS) initially introduced in 1965 by Zadeh [1]. It has been broadly using in several areas; the idea of fuzzy set is appreciated as it controls unpredictability and haziness with high accuracy. With the rapid development of society, the real world become more complex, vague, and uncertain and due to the limitation of human cognitive ability and knowledge level of human being the mean of information has also changed. Furthermore, to handle more inaccurate and unclear information in everyday life issues, several extensions of fuzzy sets are proposed by scholars in literature like intuitionistic fuzzy sets Atanassov [2] in 1986, Torra & Narukawa [27] in 2009 on hesitant fuzzy sets and decision they present an extension principle, which permits to generalize existing operations on fuzzy sets to new type of fuzzy sets and also discuss their usage in decision making, Yager and Abbasov [3] in 2013 explored Pythagorean membership grades, complex numbers, and decision making. PFSs first introduced by Yager [3] in 2013 which is highly capable than IFS to treat vagueness and inaccuracy in decision making. PFSs are categorized by degree of membership (μ), degree of non-membership (ν), and degree of hesitation (π) such that $\mu^2+\nu^2\leq 1$ or $\mu^2+\nu^2+\pi^2=1$. Zhang and Xu [4] presented an extension of TOPSIS to multiple criteria decisions making with Pythagorean fuzzy sets. Peng and Yang [5] explored some results for Pythagorean fuzzy sets. Bookstein et al. [6] presented Fuzzy Hamming distance, a new dissimilarity measure. Applications of the Fuzzy ELECTRE Method for Decision Support Systems of Cement Vendor Selection presented by Komsiyaha et al. [23], multi-criteria group decision making based on ELECTRE method in Pythagorean fuzzy information is presented by Akram et al. [24] and proposed that PFSs are extensions of IFSs. For instance, the situation with the numbers $\mu=0.86603$ and $\nu=0.5$ in such situations IFSs cannot be used but PFSs can handle this situation amicably. This is because $\mu+\nu=1.36603>1$. On the other hand, we can use PFSs as $\mu^2=0.75$ and $\nu^2=0.25$, because the condition $\mu^2+\nu^2=1$ is fulfilled. Thus, we may intuitionally say that PFSs are much wider than IFSs in tackling daily life problems. A new similarity measure for Pythagorean fuzzy sets are presented by Firozja et al. [25], and distance and similarity measures of Pythagorean fuzzy sets and their applications to multiple criteria group decision making by Zeng et al. [26].

This paper is structured as follows: In section 2, we review some preliminaries of fuzzy set, its generalization and distance and similarity measures of IFS. In section 3, we suggest several novel distance and similarity measures between PFSs with construction of axiomatic definition. In Section 4, we illustrate some examples for comparison analysis using proposed methods and existing ones. In Section 5, we construct a novel Pythagorean ELECTRE based on the proposed distance and similarity measures. We then apply the proposed method to multi-criterion decision making for ranking of alternatives in preferred order. Finally, we state our conclusion in Section 6.

2. Preliminaries

In this section, we discuss some primary concepts of intuitionistic fuzzy set and Pythagorean fuzzy set respectively.

Definition 1. An intuitionistic fuzzy set $\tilde{P}$ in Y given by Atanassov [2] as

$$\tilde{P} = \left\{ \left\langle y, \mu_{\tilde{P}}(y), \nu_{\tilde{P}}(y) \right\rangle : y \in Y \right\}$$

where $\mu_{\tilde{P}}(y): Y \to [0,1]$ is called degree of membership and $\mu(y): Y \to [0,1]$ is called degree of non-membership with the given condition $0 \le \mu_{\tilde{P}}(y) + \nu_{\tilde{P}}(y) \le 1$. The hesitation degree $\pi_{\tilde{P}}(y)$ of $\tilde{P}$ in Y is expressed as $\pi_{\tilde{P}}(y) + \mu_{\tilde{P}}(y) + \nu_{\tilde{P}}(y) = 1$, obviously $\pi_{\tilde{P}}(y) = 1 - \mu_{\tilde{P}}(y) - \nu_{\tilde{P}}(y)$. Greater degree of hesitancy $\pi_{\tilde{P}}(y)$ shows more hesitancy. Clearly, when $\pi_{\tilde{P}}(y) = 0, \forall\, y \in Y$ then IFSs fall off into a normal fuzzy set.

Definition 2. A Pythagorean fuzzy set $\tilde{P}$ in the Y given by Yager [3]

$$\tilde{P} = \left\{ \left\langle y, \mu_{\tilde{P}}(y), \nu_{\tilde{P}}(y) \right\rangle : y \in Y \right\}$$

Here $\mu_{\tilde{P}}(y)$ is called the membership degree and $\nu_{\tilde{P}}(y)$ is called the non-membership degree, with the given condition $0 \le \left(\mu_{\tilde{P}}(y)\right)^2 + \left(\nu_{\tilde{P}}(y)\right)^2 \le 1$. The hesitation degree $\pi_{\tilde{P}}(y)$ of P in Y can be expressed as $\pi_{\tilde{P}}(y) = \sqrt{1 - \left[\left(\mu_{\tilde{P}}(y)\right)^2 + \left(\nu_{\tilde{P}}(y)\right)^2\right]}$ and $\pi_{\tilde{P}}(y) \in [0,1]$ which follows:

$\left(\mu_{\tilde{P}}(y)\right)^2+\left(\nu_{\tilde{P}}(y)\right)^2+\left(\pi_{\tilde{P}}(y)\right)^2=1$. If $\left(\pi_{\tilde{P}}(y)\right)^2=0$ then $\left(\mu_{\tilde{P}}(y)\right)^2+\left(\nu_{\tilde{P}}(y)\right)^2=1$. We represent the set of all PFSs in Y by PFSs(Y), an interval-valued fuzzy set (IVFS) $\tilde{R}$ in Y, the membership degree of $y\in Y$ is $\left[\mu^2_{\tilde{R}L}(\dot{z}),\mu^2_{\tilde{R}U}(\dot{z})\right]$.

Definition 3. [3, 4]. Assume that $\tilde{P}$ and $\tilde{Q}$ are two PFSs in Y then

(i) $\tilde{P}\subseteq\tilde{Q}$ iff $\forall\ y\in Y, \mu_{\tilde{P}}(y)\le\mu_{\tilde{Q}}(y)$ and $\nu_{\tilde{P}}(y)\ge\nu\ (y)$;

(ii) $\tilde{P}=\tilde{Q}$ iff $\forall\ y\in Y, \mu_{P}(y)=\mu_{\tilde{Q}}(y)$ and $\forall\ y\in Y$, $\nu_{\tilde{Q}}(y)=\nu\ (y)$;

(iii) $\tilde{P}^c=\left\{\left\langle y,\nu_{\tilde{P}}(y),\mu_{\tilde{P}}(y)\right\rangle : y\in Y\right\}$, where, $\tilde{P}^c$ is called as the complement of PFS $\tilde{P}$.

Definition 4. [8]. Two IVFSs $s=\langle\mu_s,\nu_s\rangle$, $t=\langle\mu_t,\nu_t\rangle$ may be classified by the partial order such as $s\le t \Leftrightarrow \mu_s\le\mu_t, \nu_s\ge\nu_t$.

On the basis of partial order we get the smallest IVF as $\langle 0,1\rangle$ in the space of IVF represented by 0, and biggest IVF as $\langle 1,0\rangle$ in the space of IVF represented by 1.

Definition 5. [6]. A function $\tilde{d}: PFS(Y)\times PFS(Y) \square [0,1]$, $\tilde{d}$ is called a distance if it fulfills the following axioms:

$(\tilde{d}_1)$ $0\le\tilde{d}(\tilde{P},\tilde{Q})\le 1$;

$(\tilde{d}_2)$ $\tilde{d}(\tilde{P},\tilde{Q})=\tilde{d}(\tilde{Q},\tilde{P})$;

$(\tilde{d}_3)$ $\tilde{d}(\tilde{P},\tilde{Q})=0$ iff $\tilde{P}=\tilde{Q}$;

$(\tilde{d}_4)$ $\tilde{d}(\tilde{P},\tilde{P}^c)=1$ iff $\tilde{P}$ is a crisp set;

$(\tilde{d}_5)$ If $\tilde{P}\subseteq\tilde{Q}\subseteq\tilde{R}$, then $\tilde{d}(\tilde{P},\tilde{R})\ge\tilde{d}(\tilde{P},\tilde{Q})$ and $\tilde{d}(\tilde{P},\tilde{R})\ge\tilde{d}(\tilde{Q},\tilde{R})$;

Definition 6. [7]. Suppose a function $\tilde{S}: PFS(Y)\times PFS(Y) \square [0,1]$, $\tilde{S}$ is called a similarity measure between two PFS $\tilde{P}$ and $\tilde{Q}$ if it fulfills the following axioms:

(S_1) $0\le\tilde{S}\left(\tilde{P},\tilde{Q}\right)\le 1$;

(S_2) $\tilde{S}(\tilde{P},\tilde{Q}) = \tilde{S}(\tilde{Q},\tilde{P})$;

(S_3) $S(\tilde{P},\tilde{Q}) = 1 \Leftrightarrow \tilde{P} = \tilde{Q}$;

(S_4) If $\tilde{P} \subseteq \tilde{Q}$ and $\tilde{Q} \subseteq \tilde{R}$, then $\tilde{S}(\tilde{P},\tilde{R}) \leq min\{\tilde{S}(\tilde{P},\tilde{Q}), \tilde{S}(\tilde{Q},\tilde{R})\}$.

We review the existing distance and similarity measures of PFSs. Suppose $\tilde{P}$ and $\tilde{Q}$ be two PFSs in Y where $Y = \{y_1, y_2, y_3, ..., y_n\}$ then $\tilde{D}(\tilde{P},\tilde{Q})$ is the distance measure between two PFSs $\tilde{P}$ and $\tilde{Q}$ in Y.

The normalized Hamming distance measure is given by [29]

$$\tilde{D}_{\mathrm{Hm}}(\tilde{P},\tilde{Q}) = \frac{1}{2n}.\sum_{i=1}^{n}\left(\left|\mu_{\tilde{P}}^2(y_i) - \mu_{\tilde{Q}}^2(y_i)\right| + \left|\nu_{\tilde{P}}^2(y_i) - \nu_{\tilde{Q}}^2(y_i)\right| + \left|\pi_{\tilde{P}}^2(y_i) - \pi_{\tilde{Q}}^2(y_i)\right|\right) \tag{1}$$

The normalized Euclidean distance measure between PFSs $\tilde{P}$ and $\tilde{Q}$ in Y is suggested by [29]

$$\tilde{D}_{\mathrm{E}}(\tilde{P},\tilde{Q}) = \sqrt{\frac{1}{2n}\left(\left(\mu_{\tilde{P}}^2(y_i) - \mu_{\tilde{Q}}^2(y_i)\right)^2 + \left(\nu_{\tilde{P}}^2(y_i) - \nu_{\tilde{Q}}^2(y_i)\right)^2 + \left(\pi_{\tilde{P}}^2(y_i) - \pi_{\tilde{Q}}^2(y_i)\right)^2\right)} \tag{2}$$

Li et al. [9] proposed similarity measure $\mathrm{S_L}$ as follows:

$$S_{\mathrm{L}}(\tilde{P},\tilde{Q}) = 1 - \tilde{D}(\tilde{P},\tilde{Q}) = 1 - \sqrt{\frac{\sum_{i=1}^{n}\left(\left(\mu_{\tilde{P}}(y_i) - \mu_{\tilde{Q}}(y_i)\right)^2 + \left(\nu_{\tilde{P}}(y_i) - \nu_{\tilde{Q}}(y_i)\right)^2\right)}{2n}} \tag{3}$$

Chen [10] proposed similarity measure S_C as follows:

$$S_C = 1 - \frac{\sum_{i=1}^{n}\left|\mu_{\tilde{P}}(y_i) - \nu_{\tilde{P}}(y_i) - \left(\mu_{\tilde{Q}}(y_i) - \nu_{\tilde{Q}}(y_i)\right)\right|}{2n} \tag{4}$$

Hung and Yang [11] proposed the following similarity measures

$$S_{HY1}(\tilde{P},\tilde{Q}) = 1 - \frac{\sum_{i=1}^{n}\max\left(\left|\mu_{\tilde{P}}(y_i) - \mu_{\tilde{Q}}(y_i)\right| . \left|\nu_{\tilde{P}}(y_i) - \nu_{\tilde{Q}}(y_i)\right|\right)}{n}, \tag{5}$$

$$S_{HY_2}\left(\tilde{P},\tilde{Q}\right)=\frac{e^{S_{HY_1}\left(\tilde{P},\tilde{Q}\right)}-e^{-1}}{1-e^{-1}} \text{ and} \quad (6)$$

$$S_{HY_3}\left(\tilde{P},\tilde{Q}\right)=\frac{S_{HY_1}\left(\tilde{P},\tilde{Q}\right)}{2-S_{HY_1}\left(\tilde{P},\tilde{Q}\right)}. \quad (7)$$

Hong and Kim [12] proposed the following similarity measure

$$S_{HK}\left(\tilde{P},\tilde{Q}\right)=1-\frac{\sum_{i=1}^{n}\left(\left|\mu_{\tilde{P}}(y_i)-\mu_{\tilde{Q}}(y_i)\right|+\left|\nu_{\tilde{P}}(y_i)-\nu_{\tilde{Q}}(y_i)\right|\right)}{2n}. \quad (8)$$

Li and Cheng [13] proposed Pythagorean fuzzy similarity measure as follows:

$$S_{LC}\left(\tilde{P},\tilde{Q}\right)=1-\sqrt[q]{\frac{\sum_{i=1}^{n}\left|\psi_{\tilde{P}}(y_i)-\psi_{\tilde{Q}}(y_i)\right|}{n}} \quad (9)$$

where

$\psi_{\tilde{P}}(y_i)=\frac{\mu_{\tilde{P}}(y_i)+1-\nu_{\tilde{P}}(y_i)}{2}, \psi_{\tilde{Q}}(y_i)=\frac{\mu_{\tilde{Q}}(y_i)+1-\nu_{\tilde{Q}}(y_i)}{2}$ and $1\le q<\infty$.

Li and Xu [14] proposed Pythagorean fuzzy similarity measure $S_{LX}\left(\tilde{P},\tilde{Q}\right)$ as

$$S_{LX}\left(\tilde{P},\tilde{Q}\right)=1-\frac{\sum_{i=1}^{n}\left(\left|\mu_{\tilde{P}}(y_i)-\nu_{\tilde{P}}(y_i)-\left(\mu_{\tilde{Q}}(y_i)-\nu_{\tilde{Q}}(y_i)\right)\right|+\left|\mu_{\tilde{P}}(y_i)-\mu_{\tilde{Q}}(y_i)\right|+\left|\nu_{\tilde{P}}(y_i)-\nu_{\tilde{Q}}(y_i)\right|\right)}{4n} \quad (10)$$

Wei and Wei [15] proposed Pythagorean fuzzy similarity measure as follows:

$$S_{W}\left(\tilde{P},\tilde{Q}\right)=\frac{1}{n}\sum_{i=1}^{n}\frac{\mu_{\tilde{P}}^{2}(y_i)\mu_{\tilde{Q}}^{2}(y_i)+\nu_{\tilde{P}}^{2}(y_i)\nu_{\tilde{Q}}^{2}(y_i)}{\sqrt{\mu_{\tilde{P}}^{4}(y_i)+\nu_{\tilde{P}}^{4}(y_i)}\sqrt{\mu_{\tilde{Q}}^{4}(y_i)+\nu_{\tilde{Q}}^{4}(y_i)}} \quad (11)$$

Liang and Shi [16] proposed the following Pythagorean fuzzy similarity measure

$$S_{LS1}\left(\tilde{P},\tilde{Q}\right)=1-\sqrt[q]{\frac{\sum_{i=1}^{n}\left|\phi_{\mu}(y_i)+\phi_{\nu}(y_i)\right|}{n}}, \quad (12)$$

$$S_{LS2}\left(\tilde{P},\tilde{Q}\right)=1-\sqrt[q]{\frac{\sum_{i=1}^{n}\left|\phi_{\mu}\left(y_i\right)+\phi_{\nu}\left(y_i\right)\right|}{n}} \text{ and} \tag{13}$$

$$S_{LS3}\left(\tilde{P},\tilde{Q}\right)=1-\sqrt[q]{\frac{\sum_{i=1}^{n}\left(\eta_1\left(y_i\right)+\eta_2\left(y_i\right)+\eta_3\left(y_i\right)\right)^q}{3n}}. \tag{14}$$

Where $\phi_{\mu}\left(y_i\right)=\frac{\left|\mu_{\tilde{P}}\left(y_i\right)-\mu_{\tilde{Q}}\left(y_i\right)\right|}{2}, \phi_{\nu}\left(y_i\right)=\frac{\left|\nu_{\tilde{P}}\left(y_i\right)-\nu_{\tilde{Q}}\left(y_i\right)\right|}{2}, \phi_{\mu}\left(y_i\right)=\frac{\left|m_{\tilde{P}_1}\left(y_i\right)-m_{\tilde{Q}_1}\left(y_i\right)\right|}{2}$

$\phi_{\nu}\left(y_i\right)=\frac{\left|m_{\tilde{P}_2}\left(y_i\right)-m_{\tilde{Q}_2}\left(y_i\right)\right|}{2}, m_{\tilde{P}_1}\left(y_i\right)=\frac{\left|\mu_{\tilde{P}}\left(y_i\right)+m_{\tilde{P}}\left(y_i\right)\right|}{2}, m_{\tilde{Q}_1}\left(y_i\right)=\frac{\left|\mu_{\tilde{Q}}\left(y_i\right)+m_{\tilde{Q}}\left(y_i\right)\right|}{2}$

$m_{\tilde{P}_2}\left(y_i\right)=\frac{\left|1-\nu_{\tilde{P}}\left(y_i\right)+m_{\tilde{P}}\left(y_i\right)\right|}{2}, m_{\tilde{Q}_2}\left(y_i\right)=\frac{\left|1-\nu_{\tilde{Q}}\left(y_i\right)+m_{\tilde{Q}}\left(y_i\right)\right|}{2},$

$m_{\tilde{P}}\left(y_i\right)=\frac{\left|\mu_{\tilde{P}}\left(y_i\right)+1-\nu_{\tilde{P}}\left(y_i\right)\right|}{2}, m_{\tilde{Q}}\left(y_i\right)=\frac{\left|\mu_{\tilde{Q}}\left(y_i\right)+1-\nu_{\tilde{Q}}\left(y_i\right)\right|}{2},$

$\eta_1\left(y_i\right)=\frac{\left|\mu_{\tilde{P}}\left(y_i\right)-\mu_{\tilde{Q}}\left(y_i\right)\right|+\left|\nu_{\tilde{P}}\left(y_i\right)-\nu_{\tilde{Q}}\left(y_i\right)\right|}{2}, \eta_2\left(y_i\right)=\frac{\left|\left(\mu_{\tilde{P}}\left(y_i\right)-\nu_{\tilde{P}}\left(y_i\right)\right)-\left(\mu_{\tilde{Q}}\left(y_i\right)-\nu_{\tilde{Q}}\left(y_i\right)\right)\right|}{2},$

$\eta_3\left(y_i\right)=max\left(\frac{\pi_{\tilde{P}}\left(y_i\right)}{2},\frac{\pi_{\tilde{Q}}\left(y_i\right)}{2}\right)-min\left(\frac{\pi_{\tilde{P}}\left(y_i\right)}{2},\frac{\pi_{\tilde{Q}}\left(y_i\right)}{2}\right).$

Mitchell [17] proposed Pythagorean fuzzy similarity measure as follows:

$$S_M\left(\tilde{P},\tilde{Q}\right)=\frac{\rho_{\mu}\left(\tilde{P},\tilde{Q}\right)+\rho_{\nu}\left(\tilde{P},\tilde{Q}\right)}{2} \tag{15}$$

where $\rho_{\mu}\left(\tilde{P},\tilde{Q}\right)=1-\sqrt[q]{\frac{\sum_{i=1}^{m}\left|\mu_{\tilde{P}}\left(y_i\right)-\mu_{\tilde{Q}}\left(y_i\right)\right|^q}{n}}$, $\rho_{\nu}\left(\tilde{P},\tilde{Q}\right)=1-\sqrt[q]{\frac{\sum_{i=1}^{m}\left|\nu_{\tilde{P}}\left(y_i\right)-\nu_{\tilde{Q}}\left(y_i\right)\right|^q}{n}}$ and $1\le q<\infty$.

The similarity measure on PFS is given by Ye [18] as follows:

$$S_Y\left(\tilde{P},\tilde{Q}\right)=\frac{1}{n}\sum_{i=1}^{n}\frac{\mu_{\tilde{P}}\left(y_i\right)\mu_{\tilde{Q}}\left(y_i\right)+\nu_{\tilde{P}}\left(y_i\right)\nu_{\tilde{Q}}\left(y_i\right)}{\sqrt{\mu_{\tilde{P}}^2\left(y_i\right)+\nu_{\tilde{P}}^2\left(y_i\right)}\sqrt{\mu_{\tilde{Q}}^2\left(y_i\right)+\nu_{\tilde{Q}}^2\left(y_i\right)}}. \tag{16}$$

The similarity measure suggested by Zhang [19] as follows:

$$S_Z\left(\tilde{P},\tilde{Q}\right)=\frac{1}{n}\sum_{i=1}^{n}\frac{\left|\mu_{\tilde{P}}^2\left(y_i\right)-\nu_{\tilde{Q}}^2\left(y_i\right)\right|+\left|\nu_{\tilde{P}}^2\left(y_i\right)-\mu_{\tilde{Q}}^2\left(y_i\right)\right|+\left|\pi_{\tilde{P}}^2\left(y_i\right)-\pi_{\tilde{Q}}^2\left(y_i\right)\right|}{\left\{\begin{array}{l}\left|\mu_{\tilde{P}}^2\left(y_i\right)-\mu_{\tilde{Q}}^2\left(y_i\right)\right|+\left|\nu_{\tilde{P}}^2\left(y_i\right)-\nu_{\tilde{Q}}^2\left(y_i\right)\right|+\left|\pi_{\tilde{P}}^2\left(y_i\right)-\pi_{\tilde{Q}}^2\left(y_i\right)\right|+\left|\mu_{\tilde{P}}^2\left(y_i\right)-\nu_{\tilde{Q}}^2\left(y_i\right)\right|+\\ \left|\nu_{\tilde{P}}^2\left(y_i\right)-\mu_{\tilde{Q}}^2\left(y_i\right)\right|+\left|\pi_{\tilde{P}}^2\left(y_i\right)-\pi_{\tilde{Q}}^2\left(y_i\right)\right|\end{array}\right\}} \tag{17}$$

Peng et al. [20] proposed the following similarity measures:

$$S_{P_1}\left(\tilde{P},\tilde{Q}\right)=1-\frac{\sum_{i=1}^{n}\left|\mu_{\tilde{P}}^2\left(y_i\right)-\nu_{\tilde{P}}^2\left(y_i\right)-\left(\mu_{\tilde{Q}}^2\left(y_i\right)-\nu_{\tilde{Q}}^2\left(y_i\right)\right)\right|}{2n}, \tag{18}$$

$$S_{P_2}\left(\tilde{P},\tilde{Q}\right)=\frac{1}{n}\frac{\sum_{i=1}^{n}\mu_{\tilde{P}}^2\left(y_i\right)\wedge\mu_{\tilde{Q}}^2\left(y_i\right)+\left(\nu_{\tilde{P}}^2\left(y_i\right)\wedge\nu_{\tilde{Q}}^2\left(y_i\right)\right)}{\mu_{\tilde{P}}^2\left(y_i\right)\vee\mu_{\tilde{Q}}^2\left(y_i\right)+\left(\nu_{\tilde{P}}^2\left(y_i\right)\vee\nu_{\tilde{Q}}^2\left(y_i\right)\right)} \text{ and} \tag{19}$$

$$S_{P_3}\left(\tilde{P},\tilde{Q}\right)=\frac{1}{n}\frac{\sum_{i=1}^{n}\left(\mu_{\tilde{P}}^2\left(y_i\right)\wedge\mu_{\tilde{Q}}^2\left(y_i\right)\right)+\left(1-\nu_{\tilde{P}}^2\left(y_i\right)\right)\wedge\left(1-\nu_{\tilde{Q}}^2\left(y_i\right)\right)}{\mu_{\tilde{P}}^2\left(y_i\right)\vee\mu_{\tilde{Q}}^2\left(y_i\right)+\left(1-\nu_{\tilde{P}}^2\left(y_i\right)\right)\vee\left(1-\nu_{\tilde{Q}}^2\left(y_i\right)\right)}. \tag{20}$$

Boran and Akay [21] suggested the following similarity measure as follows:

$$S_{BA}\left(\tilde{P},\tilde{Q}\right)=1-\sqrt[q]{\frac{\sum_{i=1}^{n}\left\{\left|t\left(\mu_{\tilde{P}}\left(y_i\right)-\mu_{\tilde{Q}}\left(y_i\right)\right)-\left(\nu_{\tilde{P}}\left(y_i\right)-\nu_{\tilde{Q}}\left(y_i\right)\right)\right|^q+\left|t\left(\nu_{\tilde{P}}\left(y_i\right)-\nu_{\tilde{Q}}\left(y_i\right)\right)-\left(\mu_{\tilde{P}}\left(y_i\right)-\mu_{\tilde{Q}}\left(y_i\right)\right)\right|^q\right\}}{2n\left(t+1\right)^q}}. \tag{21}$$

Similarity measure given by Peng [22] as follows:

$$S\left(\tilde{P},\tilde{Q}\right)=1-\sqrt[P]{\frac{1}{2n\left(t+1\right)^P}\sum_{i=1}^{n}\left\{\begin{array}{l}\left|\left(t+1-a\right)\left(\mu_{\tilde{P}}^2\left(y_i\right)-\mu_{\tilde{Q}}^2\left(y_i\right)\right)-a\left(\nu_{\tilde{P}}^2\left(y_i\right)-\nu_{\tilde{Q}}^2\left(y_i\right)\right)\right|^P\\ +\left|\left(t+1-b\right)\left(\nu_{\tilde{P}}^2\left(y_i\right)-\nu_{\tilde{Q}}^2\left(y_i\right)\right)-b\left(\mu_{\tilde{P}}^2\left(y_i\right)-\mu_{\tilde{Q}}^2\left(y_i\right)\right)\right|^P\end{array}\right\}}. \tag{22}$$

The above review of literature reveals that there is still need to modify the similarity measure to achieve high accuracy and precision. Therefore, in the following section, we construct a novel distance and similarity measures based on PFSs.

3. Novel Distance and Similarity Measures between PFSs

Distance and similarity measures between PFSs are very useful and effective tool to determine the degree of dissimilarity and the degree resemblance between two sets or objects. In this section, we present a novel distance and similarity measures between PFSs utilizing the concept of Pythagorean fuzzy intervals (PFIs). Assume that two PFSs $\tilde{P}$ *and* $\tilde{Q}$ in Y with $\tilde{P}=\left\{\left\langle y_i,\mu_{\tilde{P}}^2(y_i),\mu_{\tilde{P}}^2(y_i)\right\rangle : y_i \in Y\right\}$ and $\tilde{Q}=\left\{\left\langle y_i,\mu_{\tilde{Q}}^2(y_i),\mu_{\tilde{Q}}^2(y_i)\right\rangle : y_i \in Y\right\}$ respectively. We construct the PFIs utilizing membership degree $\mu_{\tilde{P}}^2(y_i)$ of y_i to $\tilde{P}$ as lower bound and set upper bound as $1-\nu_{\tilde{P}}^2(y_i)$. Further, the membership degree $\mu_{\tilde{P}}(y_i)$ of y_i to $\tilde{P}$ is $\left[\mu_{\tilde{P}}^2(y_i),1-\nu_{\tilde{P}}^2(y_i)\right]$, i=1, 2, 3,...,n. Thus the distance between two PFSs $\tilde{P}$ and $\tilde{Q}$ in the universe of discourse $Y=\{y_1,y_2,y_3,...,y_n\}$ can be measured by comparing the intervals $\left[\mu_{\tilde{P}}^2(y_i),1-\nu_{\tilde{P}}^2(y_i)\right]$ and $\left[\mu_{\tilde{Q}}^2(y_i),1-\nu_{\tilde{Q}}^2(y_i)\right]$, i=1, 2, 3,...,n.

Definition 7. [28]. Given two interval values $p=[p_1,p_2]$ and $q=[p_2,q_2]$ the distance between p and q is

$$d'(\tilde{p},\tilde{q})=\sqrt{\int\left[\left\{\frac{\tilde{p}_1+\tilde{p}_2}{2}-\frac{\tilde{q}_1+\tilde{q}_2}{2}\right\}+\left\{\frac{\tilde{p}_1-\tilde{p}_2}{2}-\frac{\tilde{q}_1-\tilde{q}_2}{2}\right\}(2t-1)\right]^2 dt}.$$

$$=\sqrt{\left\{\frac{p_1+p_2}{2}-\frac{q_1+q_2}{2}\right\}^2+\left\{\frac{p_1-p_2}{2}-\frac{q_1-q_2}{2}\right\}^2}.$$

Thus $\forall i\in\{1,2,3,...,n\}$, $\tilde{P}_{y_i}=\left\langle\mu_{\tilde{P}}^2(y_i),\nu_{\tilde{P}}^2(y_i)\right\rangle$ and $\tilde{Q}_{y_i}=\left\langle\mu_{\tilde{Q}}^2(y_i),\nu_{\tilde{Q}}^2(y_i)\right\rangle$, the distance between their corresponding interval values $\left[\mu_{\tilde{P}}^2(y_i),1-\nu_{\tilde{P}}^2(y_i)\right]$ and $\left[\mu^2(y),1-\nu^2(y)\right]$ can be represented as follows:

$$d'(P_{y_i},Q_{y_i})=\left(\frac{\mu_{\tilde{P}}^2(y_i)+1-\nu^2(y_i)}{2}-\frac{\mu^2(y)+1-\nu^2(y)}{2}\right)^2+\frac{1}{3}\left(\frac{2\quad 2}{2}-\frac{1-\mu_{\tilde{Q}}^2(y_i)-\nu_{\tilde{Q}}^2(y_i)}{2}\right)$$

$$\sqrt{\quad \tilde{Q}\ i \quad \tilde{Q}\ i \quad 1\ \ 1-\mu_{\tilde{P}}(y_i)-\nu_{\tilde{P}}(y_i) \quad \tilde{Q}\ i \quad \tilde{Q}\ i}$$

which can also be written as follows

$$d'\left(\tilde{P}_{y_i},\tilde{Q}_{y_i}\right)=\sqrt{\left(\frac{\mu_{\tilde{P}}^2(y_i)-\nu_{\tilde{P}}^2(y_i)}{2}-\frac{\mu_{\tilde{Q}}^2(y_i)-\nu_{\tilde{Q}}^2(y_i)}{2}\right)^2+\frac{1}{3}\left(\frac{\mu_{\tilde{P}}^2(y_i)+\nu_{\tilde{P}}^2(y_i)}{2}-\frac{\mu_{\tilde{Q}}^2(y_i)+\nu_{\tilde{Q}}^2(y_i)}{2}\right)^2}. \quad (23)$$

Since all parameters $\mu_{\tilde{P}}^2(y_i),\nu_{\tilde{P}}^2(y_i),\pi_{\tilde{P}}^2(y_i),\mu_{\tilde{Q}}^2(y_i),\nu_{\tilde{Q}}^2(y_i),\pi_{\tilde{Q}}^2(y_i)$ take values in the interval $[0,1]$, we have $-1\le\mu_{\tilde{P}}^2(y_i)-\nu_{\tilde{P}}^2(y_i)\le 1$ and $-1\le\mu_{\tilde{Q}}^2(y_i)-\nu_{\tilde{Q}}^2(y_i)\le 1$. Then, the maximum value of $d'\left(\tilde{P}_{y_i},\tilde{Q}_{y_i}\right)$ can be 1, which is achieved when $\tilde{P}_{y_i}=\langle 0,1\rangle,\tilde{Q}_{y_i}=\langle 1,0\rangle$ and $\tilde{P}_{y_i}=\langle 1,0\rangle,\tilde{Q}_{y_i}=\langle 0,1\rangle$ thus, the relation $0\le d'\left(\tilde{P}_{y_i},\tilde{Q}_{y}\right)\le 1$ can be achieved. On the basis above analysis, we can suggest a novel definition of distance measure for Pythagorean fuzzy sets. Let $\tilde{P}$ and $\tilde{Q}$ are two PFSs defined in universe of discourse $Y=\{y_1,y_2,y_3,\ldots,y_n\}$ stated as $\tilde{P}_{y_i}=\left\{\left\langle y,\mu_{\tilde{P}}^2(y_i),\nu_{\tilde{P}}^2(y_i)\right\rangle \mid y\in Y\right\}$ and $\tilde{Q}_{y_i}=\left\{\left\langle y,\mu_{\tilde{Q}}^2(y_i),\nu_{\tilde{Q}}^2(y_i)\right\rangle \mid y\in Y\right\}$, respectively. Thus the distance between $\tilde{P}_{y_i}$ and $\tilde{Q}_{y_i}$ can be calculated utilizing Eq. (23) as follows:

$$D_{HN}(P,Q)=\frac{1}{n}\sum_{i=1}^{n}\left(\frac{\mu_{\tilde{P}}^2(y_i)-\nu_{\tilde{P}}^2(y_i)}{2}-\frac{\mu_{\tilde{Q}}^2(y_i)-\nu_{\tilde{Q}}^2(y_i)}{2}\right)^2+\frac{1}{3}\left(\frac{\mu_{\tilde{P}}^2(y_i)+\nu_{\tilde{P}}^2(y_i)}{2}-\frac{\mu_{\tilde{Q}}^2(y_i)+\nu_{\tilde{Q}}^2(y_i)}{2}\right)^2. \quad (24)$$

Since distance and similarity are dual concepts, thus we can find the similarity between $\tilde{P}_{y_i}$ and $\tilde{Q}_{y_i}$ using the relation, similarity = 1 − distance.

$$S_{HN}\left(\tilde{P},\tilde{Q}\right)=1-\frac{1}{n}\sum_{i=1}^{n}\sqrt{\left(\frac{\mu_{\tilde{P}}^2(y_i)-\nu_{\tilde{P}}^2(y_i)}{2}-\frac{\mu_{\tilde{Q}}^2(y_i)-\nu_{\tilde{Q}}^2(y_i)}{2}\right)^2+\frac{1}{3}\left(\frac{\mu_{\tilde{P}}^2(y_i)+\nu_{\tilde{P}}^2(y_i)}{2}-\frac{\mu_{\tilde{Q}}^2(y_i)+\nu_{\tilde{Q}}^2(y_i)}{2}\right)^2}. \quad (25)$$

Theorem. Let two PFSs $\tilde{P}$ and $\tilde{Q}$ defined in a finite universe of discourse $Y=\{y_1,y_2,y_3,\ldots,y_n\}$, then the proposed distance $D_{HN}\left(\tilde{P},\tilde{Q}\right)$ between two PFSs satisfies all the axioms $(d_1)-(d_6)$ in Definition 5.

Proof. Suppose that $\tilde{P}$ and $\tilde{Q}$ are two PFSs in universe of discourse Y. First, we give the proof of axiom (d_1) of Definition 5. Since $\left(\frac{\mu_{\tilde{P}}^2(y_i)-\nu_{\tilde{P}}^2(y_i)}{2}-\frac{\mu_{\tilde{Q}}^2(y_i)-\nu_{\tilde{Q}}^2(y_i)}{2}\right)^2\ge 0$ an

$\left(\quad 2 \qquad\qquad 2 \quad \right)$

$\tilde{}\ \tilde{}\ - \sqrt{\tilde{}\ \tilde{}\ \tilde{}\ \tilde{}\ -\ \tilde{}\ \tilde{}\ \tilde{}\ \tilde{}}$

$\tilde{P}$

$\left(\frac{\mu_{\tilde{P}}^2(y_i)+\nu_{\tilde{P}}^2(y_i)}{2} \square \frac{\mu_{\tilde{Q}}^2(y_i)+\nu_{\tilde{Q}}^2(y_i)}{2}\right)^2 \geq 0$. Thus, $D_{HN}\left(\tilde{P},\tilde{Q}\right) \geq 0, \forall\, i \in \{1,2,3,\ldots,n\}$. Hence, the axiom (d_1) of Definition 5 is proved. Now, we prove (d_2) of Definition 5, if $\tilde{P} = \tilde{Q},$ then $\forall i \in \{1,2,3,\ldots,n\}$, $\mu_{\tilde{P}}^2(y_i) = \mu_{\tilde{Q}}^2(y_i)$, $\nu_{\tilde{P}}^2(y_i) = \nu_{\tilde{Q}}^2(y_i)$ implies that $D\left(\tilde{P},\tilde{Q}\right) = 0$. Thus, the property (d_2) is satisfied. We prove (d_3) of Definition 5, Since $\tilde{d}\left(\tilde{P},\tilde{Q}\right) = 0$, we can get $\frac{\mu_{\tilde{P}}^2(y_i)\square\nu_{\tilde{P}}^2(y_i)}{2} \square \frac{\mu_{\tilde{Q}}^2(y_i)\square\nu_{\tilde{Q}}^2(y_i)}{2} = 0$ and $\frac{\mu_{\tilde{P}}^2(y_i)+\nu_{\tilde{P}}^2(y_i)}{2} \square \frac{\mu_{\tilde{Q}}^2(y_i)+\nu_{\tilde{Q}}^2(y_i)}{2} = 0,$

$\forall i \in \{1,2,3,\ldots,n\}$ which can be written identically as $\mu_{\tilde{P}}^2(y_i)\square\nu_{\tilde{P}}^2(y_i) = \mu_{\tilde{Q}}^2(y_i)\square\nu_{\tilde{Q}}^2(y_i)$, $\mu_{\tilde{P}}^2(y_i)+\nu_{\tilde{P}}^2(y_i) = \mu_{\tilde{Q}}^2(y_i)+\nu_{\tilde{Q}}^2(y_i)$ implies $\mu_{\tilde{P}}^2(y_i) = \mu_{\tilde{Q}}^2(y_i)$ and $\nu_{\tilde{P}}^2(y_i) = \nu_{\tilde{Q}}^2(y_i)$. Hence, $\forall\, y_i \in Y$, $\mu_{\tilde{P}}^2(y_i) = \mu_{\tilde{Q}}^2(y_i)$ and $\nu_{\tilde{P}}^2(y_i) = \nu_{\tilde{Q}}^2(y_i)$ holds, thus $\tilde{P} = \tilde{Q}$. For (d_4) of Definition 5, let $\tilde{P}$ and $\tilde{Q}$ are two PFSs in Y, if $\tilde{P}$ is crisp then either $\tilde{P} = 0$ or $\tilde{P} = 1$, in both cases $d\left(\tilde{P},\tilde{P}^c\right) = 1.$ Thus, the property (d_4) of Definition 5 is proved. Now we give the proof of the property (d_5) of Definition 5. Considering the condition $\tilde{P} \subseteq \tilde{Q} \subseteq \tilde{R},$ we have the following inequalities, $\mu_{\tilde{P}}^2(y_i) \leq \mu_{\tilde{Q}}^2(y_i) \leq \mu_{\tilde{R}}^2(y_i)$ and $\nu_{\tilde{P}}^2(y_i) \leq \nu_{\tilde{Q}}^2(y_i) \leq \nu_{\tilde{R}}^2(y_i)$. Then, we make a function $g(s,t)$ with two variables as $g(s,t) = \left((s \square t)\square(c \square d)\right)^2 + \frac{1}{3}\left((s+t)\square(c+d)\right)^2$ where $0 \leq s \leq 1$, $0 \leq t \leq 1$, $0 \leq c \leq 1$ and $0 \leq d \leq 1$. We partially derivative the function $g(s,t)$ with respect to the variable s and t respectively, as follows:

$$\frac{\partial g}{\partial s} = \left((s \square t)\square(c \square d)\right) + \frac{2}{3}\left((s+t)\square(c+d)\right) = \frac{8}{3}(s \square c) + \frac{4}{3}(d \square t) = \frac{4}{3}\left(2(s \square c) + (d \square t)\right).$$

$$\frac{\partial g}{\partial t} = \square\left((s \square t)\square(c \square d)\right) + \frac{2}{3}\left((s+t)\square(c+d)\right) = \frac{8}{3}(t \square c) + \frac{4}{3}(c \square s) = \frac{4}{3}\left(2(t \square c) + (c \square s)\right).$$

(i) According to the condition $0 \leq c \leq s \leq 1$ and $0 \leq t \leq d \leq 1$, respectively, we have $\frac{\partial g}{\partial s} \geq 0$ and $\frac{\partial g}{\partial t} \leq 0$. Thus $g(s,t)$ is an increasing and decreasing function for variables s and t respectively. Let $c = \mu_{\tilde{P}}^2(y)$ and $d = \nu_{\tilde{P}}^2(y)$ then $c = \mu_{\tilde{P}}^2(y) \leq \mu_{\tilde{Q}}^2(y) \leq \mu_{\tilde{R}}^2(y)$ and

$d = \nu^2_{\tilde{P}}(y) \geq \nu^2_{\tilde{Q}}(y) \geq \nu^2_{\tilde{R}}(y)$ *for* $i = 1, 2, 3, \ldots, n$. Since the monotonicity of $g(s,t)$, we observe $g\left(\mu^2_{\tilde{Q}}(y), \nu^2_{\tilde{Q}}(y)\right) \leq g\left(\mu^2_{\tilde{R}}(y), \nu^2_{\tilde{R}}(y)\right)$ for all $i = 1, 2, 3, \ldots, n$. Under condition $c = \mu^2_{\tilde{P}}(y), d = \nu^2_{\tilde{P}}(y)$ for all $i = 1, 2, 3, \ldots, n$ the specified statements satisfied $\dot{d}\left(\tilde{P}, \tilde{R}\right) = \frac{1}{2n}\sum_{i=1}^{n}\sqrt{g\left(\mu^2_{\tilde{R}}(y), \nu^2_{\tilde{R}}(y)\right)}$ and $\dot{d}\left(\tilde{P}, \tilde{Q}\right) = \frac{1}{2n}\sum_{i=1}^{n}\sqrt{g\left(\mu^2_{\tilde{Q}}(y), \nu^2_{\tilde{Q}}(y)\right)}$. So, we have $\dot{d}\left(\tilde{P}, \tilde{R}\right) \geq \dot{d}\left(\tilde{P}, \tilde{Q}\right)$.

(ii) In the condition $0 \leq s \leq c \leq 1$ and $0 \leq d \leq t \leq 1$, we have $\frac{\partial g}{\partial s} \leq 0$ and $\frac{\partial g}{\partial t} \geq 0$.so $g(s,t)$ is an increasing and decreasing function for variables s and t. Let $c = \mu^2_{\tilde{R}}(y), d = \nu^2_{\tilde{R}}(y)$ then $c = \mu^2_{\tilde{P}}(y) \leq \mu^2_{\tilde{Q}}(y) \leq \mu^2_{\tilde{R}}(y)$ and since the monotonicity of $g(s,t)$, we have $g\left(\mu^2_{\tilde{Q}}(y), \nu^2_{\tilde{Q}}(y)\right) \leq g\left(\mu^2_{\tilde{R}}(y), \nu^2_{\tilde{R}}(y)\right), \forall i = 1, 2, 3, \ldots, n.$

For $c = \mu^2_{\tilde{R}}(y), d = \nu^2_{\tilde{R}}(y), \forall i = 1, 2, 3, \ldots, n$ the specified statements satisfied $\dot{d}\left(\tilde{P}, \tilde{R}\right) = \frac{1}{2n}\sum_{i=1}^{n}\sqrt{g\left(\mu^2_{\tilde{P}}(y), \nu^2_{\tilde{P}}(y)\right)}$ and $\dot{d}\left(\tilde{Q}, \tilde{R}\right) = \frac{1}{2n}\sum_{i=1}^{n}\sqrt{g\left(\mu^2_{\tilde{Q}}(y), \nu^2_{\tilde{Q}}(y)\right)}$. Thus, we obtain $\dot{d}\left(\tilde{P}, \tilde{R}\right) \geq \dot{d}\left(\tilde{Q}, \tilde{R}\right)$. From (i) and (ii) we conclude that and $\dot{d}\left(\tilde{P}, \tilde{R}\right) \geq \dot{d}\left(\tilde{Q}, \tilde{R}\right)$. Hence, the condition of property (d_5) of Definition 5 is satisfied.

$$\dot{d}\left(\tilde{P}, \tilde{R}\right) \geq \dot{d}\left(\tilde{P}, \tilde{Q}\right)$$

4. Numerical examples and comparison

In this unit, we perform comparison analysis between our proposed distance and similarity measures with previous distance and similarity measure utilizing numerical examples. We show reasonability and usefulness of our newly suggested method by stating the following examples.

Example 1. Let the feature space be represented by $Y = \{y_1, y_2\}$. Assume that there are two PFSs $\tilde{P}$ and $\tilde{Q}$ on Y as: $\tilde{P} = \{\langle y_1, 0.5, 0.4\rangle, \langle y_2, 0.6, 0.5\rangle\}$, $\tilde{Q} = \{\langle y_1, 0.6, 0.6\rangle, \langle y_2, 0.5, 0.5\rangle\}$. Suppose a PFS $\tilde{R} = \{\langle y_1, 0.5, 0.41\rangle, \langle y_2, 0.6, 0.5\rangle\}$ is given which is highly similar but not exactly same as $\tilde{R}$. We utilize the normalized Hamming distance Eq. (1) and normalized

Euclidean distance Eq. (2) respectively to calculate the distance between two PFSs as follows:

$$\tilde{D}_{\mathrm{Hm}}\left(\tilde{R},\tilde{P}\right)=0.0020,\ \tilde{D}_{\mathrm{Hm}}\left(\tilde{R},\tilde{Q}\right)=0.1030,\ \tilde{\mathrm{D}}_{\mathrm{E}}\left(\tilde{R},\tilde{P}\right)=0.0447,\ \tilde{\mathrm{D}}_{\mathrm{E}}\left(\tilde{R}\ \tilde{Q}\right)=0.1237;$$

Using our proposed distance measure Eq. (24) we get the following results:

$$D_{HN}\left(\tilde{R},\tilde{P}\right)=0.0023\ ,\ D_{HN}\left(\tilde{R},\tilde{Q}\right)=0.0799;$$

We can realize that the suggested distance measure Eq. (24) shows the minimum discrimination capability than normalized Hamming distance Eq. (1) and normalized Euclidean distance Eq. (2).

Example 2. Let the feature space be denoted by $Y=\{y_1,y_2\}$ and there are three PFSs on Y

$\tilde{P}=\{\langle y_1,0.7,0.6\rangle,\langle y_2,0.8,0.4\rangle\}$, $\tilde{Q}=\{\langle y_1,0.9,0.3\rangle,\langle y_2,0.7,0.5\rangle\}$ and

$\tilde{R}=\{\langle y_1,0.9,0.2\rangle,\langle y_2,0.7,0.5\rangle\}$.

A model to be classified is given in term of PFS $\tilde{T}=\{\langle y_1,0.7,0.7\rangle,\langle y_2,0.8,0.3\rangle\}$. Here p =1, a =1, b = 2 and t =3 for D and $\sum_{i=1}^{2} w_i=\{0.5,0.5\}$ for D_0. Based on the distance proposed by Peng [22], we get the following results as follows:

$$D\left(\tilde{T},\tilde{P}\right)=0.1936,\quad D\left(\tilde{T},\tilde{Q}\right)=0.5020,\ D\left(\tilde{T},\tilde{R}\right)=0.5111.$$

We get the following results from the distance given by Li et al. [9] as:

$$D_L\left(\tilde{T},\tilde{P}\right)=0.0738,\ D_L\left(\tilde{T},\tilde{Q}\right)=0.2786,\ D_L\left(\tilde{T},\tilde{R}\right)=0.2971.$$

Based on method suggested by Ye [18] we get the following results as:

$$D_Y\left(\tilde{T},\tilde{P}\right)=0.0042\ ,\ D_Y\left(\tilde{T},\tilde{Q}\right)=0.0698,\ D_Y\left(\tilde{T},\tilde{R}\right)=0.0952.$$

Based on the distance proposed by Nguyen et al. [21], we obtain the following results:

$$D_0\left(\tilde{T},\tilde{P}\right)=0.0948,\ D_0\left(\tilde{T},\tilde{Q}\right)=0.3899,\ D_0\left(\tilde{T},\tilde{R}\right)=0.5347.$$

Based on our proposed distance measure Eq. (24), we have the following results:

$D_{HN}\left(\tilde{T},\tilde{P}\right)=0.0577$, $D_{HN}\left(\tilde{T},\tilde{Q}\right)=0.2579$, $D_{HN}\left(\tilde{T},\tilde{R}\right)=0.2709$.

Based on the above results it can be seen that the sample $\tilde{T}$ belongs to the pattern $\tilde{P}$. The proposed distance measure together with those Ye [18], Li et al. [9], Peng [22] and Nguyen et al.[21] shows the reasonable results.

Example 3. Let a singleton feature space be denoted by $Y=\{y_1\}$ and assume that two PFSs $\tilde{P}$ and $\tilde{Q}$ on Y are given $\tilde{P}=\{\langle y_1, 0.006, 0.991\rangle\}$, $\tilde{Q}=\{\langle y_1, 0.008, 0.991\rangle\}$.

Suppose a sample $\tilde{T}$ in terms of PFS is given as $\tilde{T}=\{\langle y_1, 0.010, 0.597\rangle\}$. Here p =1, a =1, b = 2, and t = 3 for D and $w_1=1$ for D_0.

Based on the distance proposed by Peng [22] we get the following results:

$D\left(\tilde{P},\tilde{T}\right)=0.4844$, $D\left(\tilde{Q},\tilde{T}\right)=0.4844$.

Based on the distance proposed by Nguyen et al. [21], we get the following results:

$D_0\left(\tilde{P},\tilde{T}\right)=0.4651$, $D_0\left(\tilde{Q},\tilde{T}\right)=0.4651$.

Based on Ejewa [19], we get the following results:

$D_2\left(\tilde{P},\tilde{T}\right)=0.2786$, $D_2\left(\tilde{Q},\tilde{T}\right)=0.2786$, $D_3\left(\tilde{P},\tilde{T}\right)=0.3129$, $D_3\left(\tilde{Q},\tilde{T}\right)=0.3129$.

Based on our proposed distance measure Eq. (24), we get the following outcomes as follows:

$D_{HN}\left(\tilde{P},\tilde{T}\right)=0.3612$, $D_{HN}\left(\tilde{Q},\tilde{T}\right)=0.3613$.

Based on the above numerically analysis results, we observe that the methods suggested by Peng [22], Nguyen et al. [21] and Ejewa [19] respectively, could not identify the belonging of the sample $\tilde{T}$ to the patterns $\tilde{P}$ and $\tilde{Q}$ correctly. This shows that these methods are not reasonable and useful for handling PFSs in different environments. On the other hand, our proposed distance measure Eq. (24), perform better and correctly classify the sample $\tilde{T}$ belongs to the pattern $\tilde{P}$ according to the principle of minimum distance between two PFSs. Thus, we can say that our proposed method Eq. (24) is reasonable and suitable than the methods suggested by Peng [22], Nguyen et al., [21] and Ejegwa [19], respectively.

Example 4. Let the feature space be consists of singleton element $Y=\{y_1\}$ and two patterns $\tilde{P}$ and $\tilde{Q}$ in terms of PFSs on Y are given $\tilde{P}=\{\langle y_1, 0.006, 0.991\rangle\}$ and $\tilde{Q}=\{\langle y_1, 0.008, 0.991\rangle\}$. A model to be classified is given as PFSs $\tilde{T}=\{\langle y_1, 0.010, 0.597\rangle\}$. Here p = 1, a = 1, b = 2 and t = 3 for S and $w_1=1$ for S_0. Based on the similarity proposed by Peng [22], produce the following results as: $S(\tilde{P},\tilde{T})=0.5156$ and $S(\tilde{Q},\tilde{T})=$ 0.5156. The similarity proposed by Nguyen et al. [21], we get the following results: $S_0(\tilde{P},\tilde{T})=0.5349$ and $S_0(\tilde{Q},\tilde{T})=$ 0.5349.

The method suggested by Ejegwa [19], we get the following results: $S_2(\tilde{P},\tilde{T})=0.7214$, $S_2(\tilde{Q},\tilde{T})=0.7214$, $S_3(\tilde{P},\tilde{T})=0.6871$, $S_3(\tilde{Q},\tilde{T})=0.6871$. Based on our proposed similarity measure Eq. (25), we get the following results: $S_{HN}(\tilde{P},\tilde{T})=0.6388$ and $S_{HN}(\tilde{Q},\tilde{T})=0.6387$.

The results of above numerical analysis the similarity methods proposed by Peng [22] and Nguyen et al [21] could not classify the patterns $\tilde{P}$ and $\tilde{Q}$. So their results are not reasonable and appropriate. But our proposed similarity measure Eq. (25) between two PFSs correctly classify the two patterns $\tilde{P}$ and $\tilde{Q}$. Hence, the results of proposed similarity measure Eq. (25) between two PFSs are reasonable and appropriate.

5. Application in Multicriteria Decision Making with ELECTRE Method

In this section, we utilize our proposed method in an application to multi-criteria decision making with ELECTRE method to handle daily life complex issue. The objective of this study is to examine the practicality and applicability of our proposed method in managing daily life complicated issues with appropriate and reasonable solution. Now, we apply our proposed distance measure Eq. (24) together with ELECTRE method to handle daily life problem containing multi-criteria decision making process. The steps of ELECTRE method are listed as follows:

(1). Formation of alternative indicator matrix:

First step of ELECTRE method is to create an indicator matrix $Y=(y_{ij})_{m\times n}$.

(2). Construction of Normalized indicator matrix or decision matrix:

The normalized decision making matrix of preferences (y_{ij}) for m alternatives (rows) rated on n criteria (columns):

$$\tilde{Y} = \begin{bmatrix} y_{01} & y_{02} & \cdots & \cdots & \cdots & y_{0n} \\ y_{11} & y_{12} & \cdots & \cdots & \cdots & y_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ y_{m1} & y_{m2} & \cdots & \cdots & \cdots & y_{mn} \end{bmatrix},\ i = 0,1,\ 2,\ldots,\ m;\ j = 0,1,\ 2,\ldots,\ n\,.$$

(3). Construction of weights of criteria on the basis of their significance:

Using the given formula to determine the weights W_j [30]

$$W_j = \left. \frac{\left(3\mu_j^2 + \nu_j^2\right)}{2} \middle/ \sum_{j=1}^{n}\left(\frac{\left(3\mu_j^2 + \nu_j^2\right)}{2}\right) \right.$$

(4). Development of weighted normalized decision matrix $(WNDM)$:

Using the given formula to calculate WNDM [30].

$$W_j P_j = \left(1 - \left(1 - \mu_P^2\right)^{w_j}, \left(\nu_P^2\right)^{w_j}\right)$$

(5). Find the set of concordance interval set $\left(\tilde{C}_{a_1a_2}\right)$:

Let $P = \{a_1, a_2, a_3, \ldots, a_n\}$ denote a finite set of alternatives, here divide the criteria sets into two distinct sets $\tilde{C}_{a_1a_2}$ and $\tilde{D}_{a_1a_2}$.

Using the given formula to find concordance interval set $\left(\tilde{C}_{a_1a_2}\right)$ is

$$\tilde{C}_{a_1a_2} = \left\{ j \mid y_{a_1 j} \geq y_{a_2 j} \right\}$$

(6). Find the set of discordance interval sets $\left(\tilde{D}_{a_1a_2}\right)$:

Using the given formula to find discordance interval set is stated as

$$\tilde{D}_{a_1a_2} = \{ j \mid y_{a_1 j} < y_{a_2 j} \} = \tilde{J} - \tilde{C}_{a_1a_2}$$

(7). **Determine concordance interval matrix:**

$$\tilde{C}_{a_1a_2} = \sum_{j \in \tilde{C}_{a_1a_2}} W_j$$

Concordance index shows the preferred of the statement "P over Q" and it is defined as:

$$\tilde{C} = \begin{bmatrix} - & \tilde{c}(1,2) & \dots & \tilde{c}(1,n) \\ \tilde{c}(2,1) & - & \dots & \tilde{c}(2,n) \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{c}(n,1) & \tilde{c}(n,2) & \cdots & - \end{bmatrix}$$

(8). **Determine the discordance interval matrix** $(\tilde{D}_{a_1a_2})$:

Discordance index of $d(a_1, a_2)$ expressed the favorite of dissatisfaction in result of form a_1 relatively than form a_2.

$$\tilde{d}(a_1, a_2) = \frac{max_{j \in \tilde{D}_{a_1a_2}} |\tilde{v}_{a_1 j} - \tilde{v}_{a_2 j}|}{max_{j \in J, r, s \in I} |\tilde{v}_{rj} - \tilde{v}_{sj}|}$$

$$\tilde{D} = \begin{bmatrix} - & \tilde{d}(1,2) & \dots & \tilde{d}(1,n) \\ \tilde{d}(2,1) & - & \dots & \tilde{d}(2,n) \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{d}(n,1) & \tilde{d}(n,2) & \cdots & - \end{bmatrix}$$

(9). **Define the set of concordance index matrix** (CIM):

CIM may be describe as:

$$\bar{\tilde{c}} = \frac{\sum_{a_1=1}^{n} \sum_{a_2=1}^{n} \tilde{c}(a_1, a_2)}{n(n-1)}$$

where the critical value $(\bar{\tilde{c}})$ can be calculated by normal domination index. So, a Boolean matrix (E) is known as:

$$\begin{cases} e(a_1, a_2) = 1, if \tilde{c}(a_1, a_2) \geq \bar{\tilde{c}} \\ e(a_1, a_2) = 0, if \tilde{c}(a_1, a_2) < \bar{\tilde{c}} \end{cases}$$

(10). **Determine the set of discordance index matrix:**

The discordance index is defined as:

$$\bar{\tilde{d}} = \frac{\sum_{a_1=1}^{n} \sum_{a_2=1}^{n} \tilde{d}(a_1, a_2)}{n(n-1)}$$

On the basis of discordance index stated above, the discordance index matrix is known as:

$$\begin{cases} g(a_1, a_2) = 1, \ if \ \tilde{d}(a_1, a_2) \geq \bar{\tilde{d}} \\ g(a_1, a_2) = 0, if \ \tilde{d}(a_1, a_2) \leq \bar{\tilde{d}} \end{cases}$$

(11) (i) **Calculate the net superior value:**

Suppose $\tilde{c}_{a_1}$ be the following superior. $\tilde{c}_{a_1}$ together the sums of number of Reasonable dominance entire alternatives, and the greater, the superior. $\tilde{c}_{a_1}$ is known as:

$$c_{a_1} = \sum_{a_2=1}^{n} c_{(a_1, a_2)} - \sum_{a_2=1}^{n} c_{(a_2, a_1)}$$

(ii) **Determine the net inferior value:**

Suppose $\tilde{d}_a$ be the succeeding net inferior value. On contrary, it is used to calculate inferior ranking of the alternatives:

$$\tilde{d}_a = \sum_{b=1}^{n} \tilde{d}_{(a,b)} - \sum_{b=1}^{n} \tilde{d}_{(b,a)}$$

Examples 3. Mobile phones have become an essential part of our daily life as they offer convenience and connectivity on-the-go. They come in various shapes, sizes, and specifications, and can be customized to suit the user's needs and preferences. There are many different brands of mobile phones on the market today with different specifications. So it is much difficult for a customer to decide which mobile phone is better for use.

Consider a buyer who wants to buy a mobile. Let four types of smart phones are considered as alternatives $\tilde{R}_1, \tilde{R}_2, \tilde{R}_3$ and $\tilde{R}_4$ be available. The buyer takings into consideration four attributes to choose which smart phone is appropriate to buy under the following four criteria's:

$\tilde{S}_1$: Battery timing, $\tilde{S}_2$: Internal storage, $\tilde{S}_3$: Price, $\tilde{S}_4$: Camera result.

Decision Goal: To buy a mobile.

Criteria: Battery timing, Price, Internal storage and Camera result.

Alternatives: $\tilde{R}_1, \tilde{R}_2, \tilde{R}_3$ and $\tilde{R}_4$.

Method to be used: ELECTRE-1

We note that $\tilde{S}_3$ is cost attribute while other three are benefit attributes. The values assign by decision makers are specified in the following Table1.

Table 1. Alternative indicator matrix

	$\tilde{S}_1$	$\tilde{S}_2$	$\tilde{S}_3$	$\tilde{S}_4$
$\tilde{R}_1$	$(0.6, 0.5)$	$(0.3, 0.7)$	$(0.3, 0.6)$	$(0.7, 0.6)$
$\tilde{R}_2$	$(0.3, 0.7)$	$(0.4, 0.5)$	$(0.2, 0.4)$	$(0.3, 0.8)$
$\tilde{R}_3$	$(0.4, 0.2)$	$(0.7, 0.4)$	$(0.2, 0.8)$	$(0.7, 0.4)$
$\tilde{R}_4$	$(0.6, 0.6)$	$(0.8, 0.3)$	$(0.1, 0.8)$	$(0.7, 0.5)$

Table 1, shows the evaluations of each alternatives $\tilde{R}_i$ over the criteria $\tilde{S}_j$.

Since the criteria $\tilde{S}_3$ is cost attribute so we have to convert it into the benefit criteria by taking the complement of $\tilde{S}_3$ as follows:

$$\tilde{S}_3' = \{ (0.6,0.3),(0.4,0.2),(0.8,0.2),(0.8,0.1)\}$$

Table 2. Pythagorean fuzzy decision matrix

	$\tilde{S}_1$	$\tilde{S}_2$	$\tilde{S}_3$	$\tilde{S}_4$
$\tilde{R}_1$	$(0.6, 0.5)$	$(0.3, 0.7)$	$(0.6, 0.3)$	$(0.7, 0.6)$
$\tilde{R}_2$	$(0.3, 0.7)$	$(0.4, 0.5)$	$(0.4, 0.2)$	$(0.3, 0.8)$
$\tilde{R}_3$	$(0.4, 0.2)$	$(0.7, 0.4)$	$(0.8, 0.2)$	$(0.7, 0.4)$
$\tilde{R}_4$	$(0.6, 0.6)$	$(0.8, 0.3)$	$(0.8, 0.1)$	$(0.7, 0.5)$

Table 2, shows the Pythagorean fuzzy decision matrix of each alternatives $\tilde{R}_i$ over the criteria $\tilde{S}_j$.

We obtained the following weights using step#3.

$$w = \{0.1942, 0.2460, 0.2676, 0.2921\}$$

Normalized weighted decision matrix is constructed using step#4

Table 3. Weighted normalized decision matrix

	$\tilde{S}_1$	$\tilde{S}_2$	$\tilde{S}_3$	$\tilde{S}_4$
$\tilde{R}_1$	$(0.0830,0.7640)$	$(0.0229,0.8391)$	$(0.1126,0.5250)$	$(0.1785,0.7420)$
$\tilde{R}_2$	$(0.0181,0.8706)$	$(0.0420,0.7110)$	$(0.0456,0.4226)$	$(0.0272,0.8778)$
$\tilde{R}_3$	$(0.0333,0.5352)$	$(0.1527,0.6371)$	$(0.2392,0.4226)$	$(0.1785,0.5855)$
$\tilde{R}_4$	$(0.0830,0.8200)$	$(0.2222,0.5530)$	$(0.2392,0.2916)$	$(0.1785,0.6670)$

The concordance interval sets are constructed using step #5.

$C_{12} = [\tilde{S}_1, \tilde{S}_3, \tilde{S}_4]$, $C_{21} = [\tilde{S}_2]$, $C_{31} = [\tilde{S}_2, \tilde{S}_3, \tilde{S}_4]$, $C_{41} = [\tilde{S}_1, \tilde{S}_2, \tilde{S}_3, \tilde{S}_4]$,

$C_{13} = [\tilde{S}_1, \tilde{S}_4]$, $C_{23} = [0]$, $C_{32} = [\tilde{S}_1, \tilde{S}_2, \tilde{S}_3, \tilde{S}_4]$, $C_{42} = [\tilde{S}_1, \tilde{S}_2, \tilde{S}_3, \tilde{S}_4]$,

$C_{24} = [0]$, $C_{34} = [\tilde{S}_3, \tilde{S}_4]$, $C_{43} = [\tilde{S}_1, \tilde{S}_2, \tilde{S}_3, \tilde{S}_4]$.

The value of concordance interval sets is listed in the following Table 4.

Table 4. Concordance interval sets

	$\tilde{S}_1$	$\tilde{S}_2$	$\tilde{S}_3$	$\tilde{S}_4$
$\tilde{R}_{12}$	1	0	1	1
$\tilde{R}_{13}$	1	0	0	1
$\tilde{R}_{14}$	1	0	0	1
$\tilde{R}_{21}$	0	1	0	0
$\tilde{R}_{23}$	0	0	0	0
$\tilde{R}_{24}$	0	0	0	0
$\tilde{R}_{31}$	0	1	1	1
$\tilde{R}_{32}$	1	1	1	1
$\tilde{R}_{34}$	0	0	1	1
$\tilde{R}_{41}$	1	1	1	1
$\tilde{R}_{42}$	1	1	1	1
$\tilde{R}_{43}$	1	1	1	1

Now, we construct the discordance interval sets utilizing step#6 as follows:

Discordance interval sets

$$D_{12} = \left[\tilde{S}_2\right], \quad D_{21} = \left[\tilde{S}_1, \tilde{S}_2, \tilde{S}_3\right], \quad D_{31} = \left[\tilde{S}_1\right], \quad D_{41} = [0],$$

$$D_{13} = \left[\tilde{S}_2, \tilde{S}_3\right], \quad D_{23} = \left[\tilde{S}_1, \tilde{S}_2, \tilde{S}_3, \tilde{S}_4\right], \quad D_{32} = [0], \quad D_{42} = [0],$$

$$D_{14} = \left[\tilde{S}_2, \tilde{S}_3\right], \quad D_{24} = \left[\tilde{S}_1, \tilde{S}_2, \tilde{S}_3, \tilde{S}_4\right], \quad D_{34} = \left[\tilde{S}_1, \tilde{S}_2\right], \quad D_{43} = [0].$$

The results of the discordance interval sets are listed in the following Table 5.

Table 5. Discordance interval sets

	$\tilde{S}_1$	$\tilde{S}_2$	$\tilde{S}_3$	$\tilde{S}_4$
$\tilde{R}_{12}$	0	1	0	0
$\tilde{R}_{13}$	0	1	1	0
$\tilde{R}_{14}$	0	1	1	0
$\tilde{R}_{21}$	1	0	1	1
$\tilde{R}_{23}$	1	1	1	1
$\tilde{R}_{24}$	1	1	1	1
$\tilde{R}_{31}$	1	0	0	0
$\tilde{R}_{32}$	0	0	0	0
$\tilde{R}_{34}$	1	1	0	0
$\tilde{R}_{41}$	0	0	0	0
$\tilde{R}_{42}$	0	0	0	0
$\tilde{R}_{43}$	0	0	0	0

Numerical results of concordance interval matrix using step #7 is given in the following Table 6.

Table 6. Concordance interval matrix

	$\tilde{R}_1$	$\tilde{R}_2$	$\tilde{R}_3$	$\tilde{R}_4$	*Sum*
$\tilde{R}_1$	0	0.7539	0.4863	0.4863	1.7265
$\tilde{R}_2$	0.2460	0	0	0	0.2460
$\tilde{R}_3$	0.8057	0.9999	0	0.5597	2.3653
$\tilde{R}_4$	0.9999	0.9999	0.9999	0	2.9997
Sum	2.0516	2.7537	1.4862	1.0460	7.3375

The results of discordance interval matrix using step # 8 are displayed in the following Table 7.

Table 7. Discordance interval matrix

	$\tilde{R}_1$	$\tilde{R}_2$	$\tilde{R}_3$	$\tilde{R}_4$	*Sum*
$\tilde{R}_1$	0	0.8400	1	1	2.8400
$\tilde{R}_2$	1	0	1	1	3
$\tilde{R}_3$	0.9492	0	0	1	1.9492
R_4	0	0	0	0	0
sum	1.9492	0.8400	2	3	7.7892

The concordance index matrix can be calculated by using step # 9 as follows:

$$c_{bar} = \bar{c} = \frac{7.3375}{4(4-1)}, \qquad c_{bar} = 0.6115$$

The results of concordance index matrix are displayed in the following Table 8.

Table 8. Concordance index matrix

	$\tilde{R}_1$	$\tilde{R}_2$	$\tilde{R}_3$	$\tilde{R}_4$
$\tilde{R}_1$	0	1	0	0
$\tilde{R}_3$	0	0	0	0
$\tilde{R}_3$	1	1	0	0
R_4	1	1	1	0

The discordance index matrix can be calculated by using step # 10 as follows:

$$d_{bar} = \bar{d} = \frac{7.7892}{4(4-1)}, \quad \bar{d} = 0.6491$$

The results of discordance index matrix are displayed in the following Table 9.

Table 9. Discordance index matrix

	$\tilde{R}_1$	$\tilde{R}_2$	$\tilde{R}_3$	$\tilde{R}_4$
$\tilde{R}_1$	1	0	0	0
$\tilde{R}_3$	0	1	0	0
$\tilde{R}_3$	0	1	1	0
R_4	1	1	1	1

Next, we calculate the net superior value, utilizing step # $11(i)$ and the results which are given in the following Table 10.

Table 10. Net superior value

		$Rank$
$\tilde{R}_1$	-0.3251	3
$\tilde{R}_2$	-2.5077	4
$\tilde{R}_3$	0.8791	2
$\tilde{R}_4$	1.9537	1

Finally, we calculate the net inferior value, using step # $11(ii)$ as follows

Table 11. Net inferior value

		$Rank$
$\tilde{R}_1$	0.8908	3
$\tilde{R}_2$	2.1600	4
$\tilde{R}_3$	-0.0508	2
$\tilde{R}_4$	-2	1

On the basis of the final ranking of results from the above Table 10 and Table 11 respectively, the final ranking is made on the basis of the highest value in the Table 10 and the lowest value in the Table 11 respectively. Therefore, $\tilde{R}_4$ can be chosen as the best alternative among all given alternatives. It is seen that there is no controversy in ranking of alternatives of Table 10 and Table 11 respectively. Hence, we conclude that a customer can buy a smart phone $\tilde{R}_4$.

6. Conclusion and Discussion

In this paper we proposed novel distance and similarity measures on the basis of PFSs and utilized it in handling MCDM problems. The proposed distance and similarity measures based on Pythagorean fuzzy sets satisfied all the properties in the axiomatic definition. To check the validity of proposed method we give several numerical examples. To show the usefulness and practical applications of proposed methods, we have presented examples related to pattern recognition. Finally, we use the ELECTRE Method to solve problems related to daily life involving multi criteria decision making. We have used our method in the selection of best mobile phone. Numerical examples clarify that the proposed distance and similarity measures can handle daily life issues related to pattern recognition and MCDM respectively.

Compliance with Ethical Standards

Disclosure of potential conflicts of interest

The authors declare that they have no any conflict of interests.

Research involving human participants and/or animals

This article does not contain any studies with human participants or animals performed by any of the authors.

Data Availability Statement

All the data is included in the manuscript

References

[1] L. A. Zadeh, Fuzzy sets, "Information and Computation 8 (1965) 338–353.

[2] K. T. Atanassov, Intuitionistic fuzzy sets. Fuzzy Sets and Systems (1986) 20 87-96.

[3] R. R. Yager, A. M. Abbasov, Pythagorean membership grades, complex numbers, and decision making, International Journal of Intelligent Systems 28(5) (2013) 436–452.

[4] X. L. Zhang, Z. S. Xu, Extension of TOPSIS to multiple criteria decisions making with Pythagorean fuzzy sets, International Journal of Intelligent Systems 29(4) (2014) 1061–1078.

[5] X. D. Peng, Y. Yang, Some results for Pythagorean fuzzy sets, International Journal of Intelligent Systems 30 (2015) 1133–1160.

[6] A. Bookstein, S. T. Klein, T. Raita, Fuzzy Hamming distance: a new dissimilarity measure, Paper presented at the Annual Symposium on Combinatorial Pattern Matching, 2001.

[7] X. Peng H. Yuan, Y. Yang, Pythagorean fuzzy information measures and their applications, International Journal of Intelligent Systems 32 (2017) 991–1029.

[8] X. Pen, H. Yuan, Y. Yang, Pythagorean fuzzy information measures and their applications, Int J Intell Syst 32 (2017) 991–1029.

[9] Y. Li, D. L. Olson, Z. Qin, Similarity measures between intuitionistic fuzzy (vague) sets: a comparative analysis, Pattern Recognit Lett 28 (2007) 278–285.

[10] S. M. Chen, Similarity measures between vague sets and between elements, IEEE Trans Syst Man Cyber 27 (1997) 153–158.

[11] W. L. Hung, M. S. Yang, Similarity measures of intuitionistic fuzzy sets based on Hausdorff distance, Pattern Recognit Lett 25 (2004) 1603–1611.

[12] D. H. Hong, C. Kim, A note on similarity measures between vague sets and between elements, Inf Sci 115 (1999) 83–96.

[13] D. F. Li, C. T. Cheng, New similarity measures of intuitionistic fuzzy sets and application to pattern recognitions, Pattern Recognit Lett 23 (2002) 221–225.

[14] F. Li, Z. Y. Xu, Measures of similarity between vague sets, J Softw 12 (2001) 922–927.

[15] G. Wei, Y. Wei, Similarity measures of Pythagorean fuzzy sets based on the cosine function and their applications, Int J Intell Syst 33 (2018) 634–652.

[16] Z. Z. Liang, P. F. Shi, Similarity measures on intuitionistic fuzzy sets, Pattern Recognit Lett 24 (2003) 2687–2693.

[17] H. B. Mitchell, On the Dengfeng–Chuntian similarity measure and its application to pattern recognition, Pattern Recognit Lett 24(45) (2003) 3101–3104.

[18] J. Ye, Cosine similarity measures for intuitionistic fuzzy sets and their applications, Math Comput Model 53 (2011) 91–97.

[19] Paul Augustine Ejegwa , Distance and similarity measures for Pythagorean fuzzy sets, 2018.

[20] X. Peng, H. Yuan, Y. Yang, Pythagorean fuzzy information measures and their applications, Int J Intell Syst 32 (2017) 991–1029.

[21] Xuan Thao Nguyen, Van Dinh Nguyen, Van Dinh Nguyen, Harish Garg, Exponential similarity measures for Pythagorean fuzzy sets and their applications to pattern recognition and decision-making process, 2019

[22] Xindong Peng, New similarity measure and distance measure for Pythagorean fuzzy set, Complex & Intelligent Systems 5 (7 November, 2018) 101–111. https://doi.org/10.1007/s40747-018-0084-x

[23] Siti Komsiyaha, Rini Wongso, Syali Widia Pratiwi, Applications of the Fuzzy ELECTRE Method for Decision Support Systems of Cement Vendor Selection, (12–13 September 2019).

[24] Muhammad Akram, Farwa Ilyas, Harish Garg, Multi-criteria group decision making based on ELECTRE I method in Pythagorean fuzzy information, (11 June 2019).

[25] M. A. Firozja, B. Agheli, E. B. Jamkhaneh, A new similarity measure for Pythagorean fuzzy sets, 2019.

[26] W. Y. Zeng, D. Q. Li, Q. Yin, Distance and similarity measures of Pythagorean fuzzy sets and their applications to multiple criteria group decision making, 2018.

[27] V. Torra, Y. Narukawa, On hesitant fuzzy sets and decision in The 18th IEE international conference on fuzzy system, 2009.

[28] Y. L. Lai, T. Y. Liu, C. L. Hwang, TOPSIS for MODM, 1994.

[29] Di. Ke, Yafei. Song, Wen. Quan, New Distance Measure for Atanassov's Intuitionistic Fuzzy Sets and Its Application in Decision Making, (19 September 2018).

[30] M. S. Yang, Z. Hussain, M. Ali, Belief and Plausibility Measures on Intuitionistic Fuzzy Sets with Construction of Belief-Plausibility TOPSIS, Complexity, (2020) Volume 2020 |Article ID 7849686 | https://doi.org/10.1155/2020/7849686